Name:

Precalculus (Math 1045)
PRACTICE Final Examination

Professor Paul Bailey
December 11, 2006

The examination contains ten problems which are worth 10 points each, and two additional extra credit problems worth 10 points each.

No books, notes, calculators, or other electronic devices are permitted.

Using trigonometric identities and the following table, it is possible to accurately compute the sine, cosine, tangent, cotangent, secant, and cosecant of the angles 0°, 15°, 18°, 30°, 36°, 45°, 60°, 72°, 75°, 90°, and many others.

θ	$0°$	$45°$	$60°$	$72°$
$\cos\theta$	1	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{5}-1}{2}$

P 1	P 2	P 3	P 4	P 5	P 6	P 7	P 8	P 9	P 10	P 11	P 12	Total

Problem 1. (Sets and Functions)
Let $A = \{2, 4, 6, 8\}$, $B = \{3, 4, 5, 6, 7\}$, $C = [0, 4]$, and $D = (2, 8]$. Write each set as a union of disjoint intervals.

(a) $E = C \cup D$

(b) $F = C \setminus D$

(c) $G = E \setminus (A \cap B)$

(d) $H = G \cup B$

(e) $I = H \cap C$

Define $f : B \to \mathbb{R}$ by $f(x) = \dfrac{x^3 - 3}{x^2 - 4}$.

(f) Find the range of f.

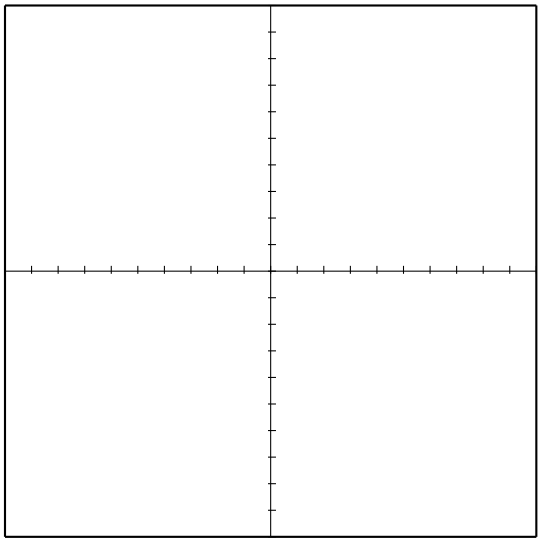
(g) Is f injective?

(h) Is f surjective?

Problem 2. (Linear Functions)
Create a linear function $f(x) = y$ by solving the equation

$$2x - 8y + 9 = 0$$

for y. Fill in the table, and use this information to sketch the graph.



Equation: $2x - 8y + 9 = 0$

Standard Form:

m: **b:**

Slope:

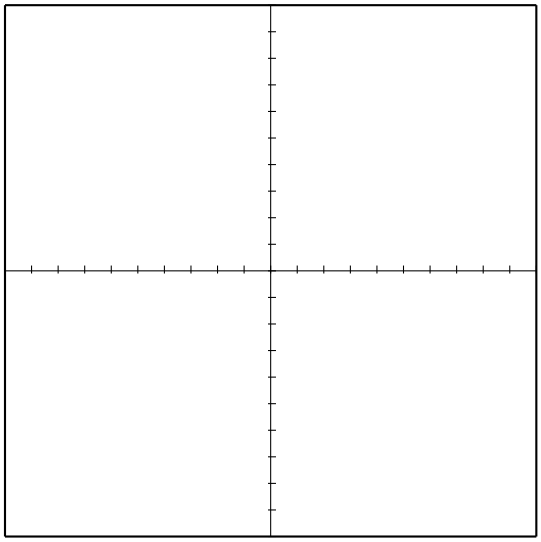
y-intercept:

x-intercept:

Problem 3. (Quadratic Functions)
Analyze the polynomial function

$$f(x) = 5x^2 - 8x + 1$$

by filling in the table below, and use this information to sketch the graph.



Equation: $y = 5x^2 - 8x + 1$

Normal Form:

Shifted Form:

a: **b:** **c:** **h:** **k:**

Discriminant:

Zeros:

y-intercept:

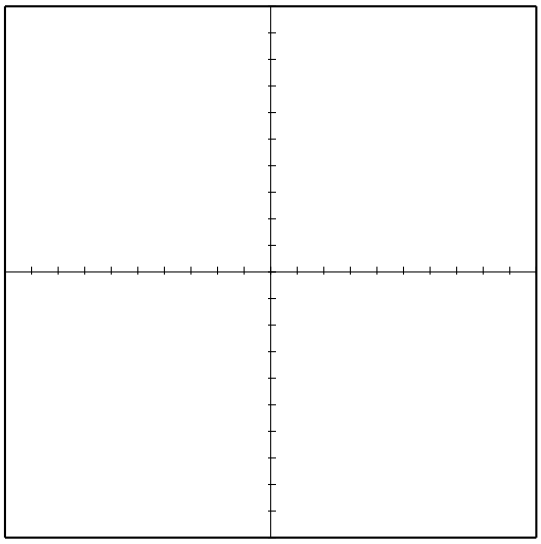
x-intercept(s):

Vertex:

Problem 4. (Polynomial Functions)
Analyze the polynomial function

$$f(x) = 2x^3 - 5x^2 + 18x - 8$$

by filling in the table below, and use this information to sketch the graph.



Polynomial: $y = 2x^3 - 5x^2 + 18x - 8$

Degree:

Leading Coefficient:

Constant Coefficient:

Zeros:

y-intercept:

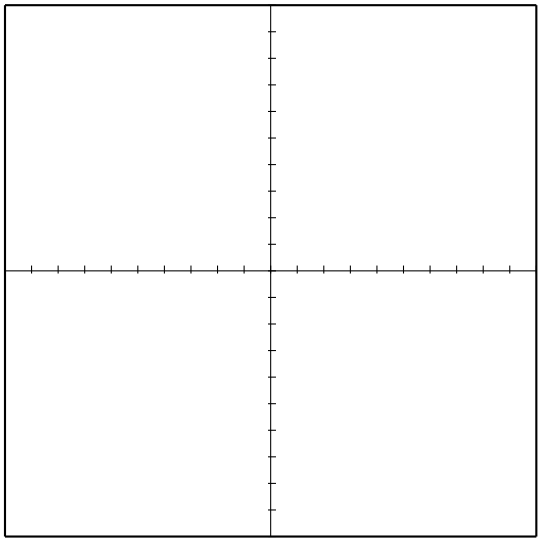
x-intercepts:

Shape:

Problem 5. (Rational Functions)
Analyze the rational function

$$f(x) = \frac{x^3 - 7x + 6}{x^2 - 16}$$

by filling in the table below, and use this information to sketch the graph.



Equation: $y = \dfrac{x^3 - 7x + 6}{x^2 - 16}$

Degree:

Zeros:

Poles:

y-intercept:

x-intercepts:

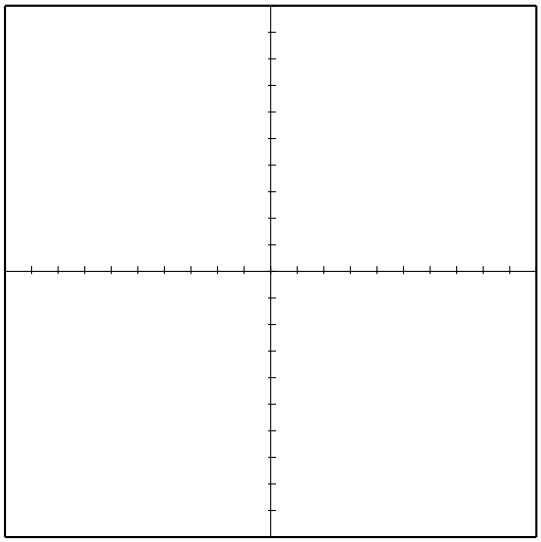
Vertical Asymptotes:

Polynomial Asymptote:

Problem 6. (Elliptical Equations)
Consider the equation of an ellipse with horizontal focal axis

$$25x^2 + y^2 = 6y + 16.$$

Put the equation in standard form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$. Identify the constants a, b, c, h, and k, where c denotes the distance from the center to a focus. Identify the center, vertices, covertices, and foci. Graph the ellipse, including all of these features.



Standard Form:

a: **b:** **c:** **h:** **k:**

Center:

Vertices:

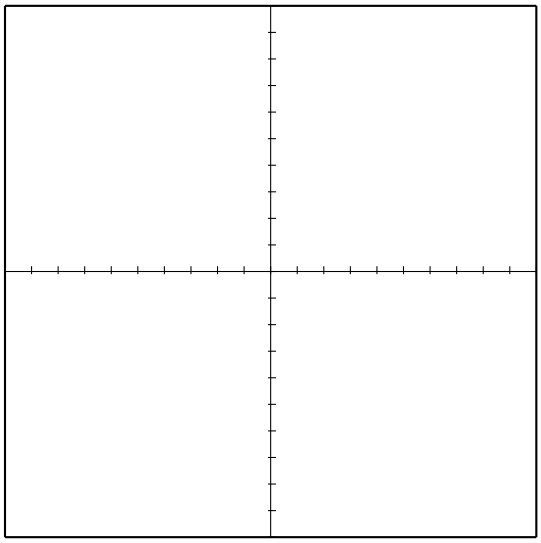
Covertices:

Foci:

Problem 7. (Hyperbolic Equations)
Consider the equation of an hyperbola with horizontal focal axis

$$9x^2 = 4y^2 + 36.$$

Put the equation in standard form $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$.. Identify the constants a, b, c, h, and k, where c denotes the distance from the center to a focus. Identify the center, vertices, covertices, foci, and asymptotes. Graph the hyperbola, including all of these features.



Standard Form:

a: **b:** **c:** **h:** **k:**

Center:

Vertices:

Covertices:

Foci:

Asymptotes:

Problem 8. (Trigonometric Functions)

(a) Find all values for x in the range $[0°, 360°)$ which satisfy the equation

$$4\cos(2\theta) = 3.$$

(b) Find all values for x in the range $[0°, 360°)$ which satisfy the equation

$$\cos^4\theta - \cos^2\theta + \frac{1}{8} = 0.$$

(c) Find an algebraic expression for the function

$$f(x) = \tan\left(\operatorname{arcsec}\left(\frac{x}{5}\right)\right).$$

Problem 9. (Solving Triangles)
Let a, b, c be the lengths of the sides of a triangle with corresponding opposite angles α, β, and γ. If $a = 4$, $b = 7$, and $\gamma = 30°$, find c.

Problem 10. (Regular Polygons)
A polygonal is *regular* if all of the sides have equal length, and all of the included angles are equal. It is *inscribed* in a circle if all of the vertices lie on the circle.

Compute the area of a regular pentagon inscribed in a circle of radius one.

Problem 11. (Vectors and Complex Numbers)

(a) Let $\vec{v} = \langle 1, t\rangle$ and $\vec{w} = \langle 5, 12\rangle$. Find y so that the projection of $\vec{v}$ onto $\vec{w}$ is 1.

(b) Find all complex numbers z such that $z^5 = 1$.

Problem 12. (Exponential and Logarithmic Functions)
Solve for x.

(a) $625^x = \frac{1}{25}$

(b) $343^{(x-1)} = \dfrac{49^{(2x-2)}}{7^{(x-3)}}$

(c) $\log_{81}\frac{1}{27} = x$

(d) $\log_{13}(3x - 5) = 3$

(e) $\log_{(x+5)}(17x + 13) = 2$

Problem 13. (Rational Functions)
The *components* of a rational function are the connected sections of the graph between the poles. Let

$$f(x) = \frac{(x+7)^3(x+4)^2(x-2)^3(x-8)^7(x-11)}{(x+5)(x+2)^5(x-5)(x-9)}.$$

(a) Is $f(7)$ positive? Is $f(0)$ positive? Is $f(-3)$ positive?

(b) Count the components of f, and say how many are above, below, or crossing the x-axis.

Problem 14. (Conics)
Find the rectangular equation of an ellipse centered at the origin passing through the points $(8, 0)$ and $(3, 4)$.

Problem 15. (Eccentricity)
Find the center of the conic $r = \frac{ed}{1+e\sin\theta}$.